

**Broadcast Capability of  
Direct-Sequence and Hybrid  
Spread Spectrum**

**By**

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# BROADCAST CAPABILITY OF DIRECT-SEQUENCE AND HYBRID SPREAD SPECTRUM

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## ABSTRACT

Two forms of spread-spectrum signaling: direct-sequence and hybrid (direct-sequence/frequency-hopped) are shown to provide high broadcast capability especially when used in conjunction with forward-error-control coding schemes. The broadcast capability is defined as the maximum number of **simultaneous distinct messages** that can be transmitted to distant receivers from a single transmitter at a given bit-error-rate. This quantity provides a useful measure of the capacity of hub-to-mobile or satellite-to-earth-station links of communication networks. When **bursty** data or voice traffic is dominant in such networks, the above forms of spread-spectrum code-division multiple-access (CDMA) provide a viable alternative to frequency-division (FDMA) or time-division (TDMA) multiple-access.

Different ways of multiplexing the direct-sequence and hybrid signals are presented which employ distinct carriers, distinct pairs of orthogonal carriers, and only two orthogonal carriers to broadcast the different messages. Systems with chip-synchronous signals and systems where random delays are introduced between the signals are considered. The average error probability of all systems is evaluated using the characteristic-function and Gaussian-approximation techniques. Besides the uncoded systems, systems using Reed-Solomon and convolutional codes are analyzed. A comparison of the broadcast capability of the different schemes is presented.



## I. INTRODUCTION

In the past ten years spread-spectrum multiple-access (SSMA) systems have received considerable attention in the literature (see references [1]-[14]; we have not attempted to compose an exhaustive list here). Besides the properties of low detectability (LPI), anti-jam resistance, and privacy, which are especially desirable in military multiple-access systems; SSMA offers (i) simultaneous channel access without the need for time coordination between the different users (ii) low peak-to-average power ratio—desirable for bursty traffic situations—and (iii) resistance to frequency-selective and multipath fading. The latter three properties of SSMA are of interest to commercial multiple-access systems like the Mobile Satellite Systems (MSAT) and Very Small Aperture Terminal Satellite Systems (VSAT).

Two modes of operation of SSMA have attracted most of the attention: the **interference channel mode** (several point-to-point or paired transmissions) (see [1]-[10]) where each receiver despreads and demodulates one of the transmitted signals (using distinct spread-spectrum codes) without any cooperation from other receivers; and the **multiple-access mode** (multipoint-to-point) where several stations communicate with a single receiver which either (i) employs matched filters to despread and demodulate all or some of the transmitted signals separately (suboptimal multi-receiver) (see [11]-[12]) or (ii) does the same with some degree of cooperation between the different matched filters (optimal or near-optimal multi-receiver) see [13]-[14]. For both modes, the **multiple-access capability** of the spread-spectrum system is defined as the maximum number of simultaneous distinct transmitted signals from independent stations that can be tolerated in the neighborhood of a receiver (single or multi-receiver) so that the error probability for the reception of a particular signal does not exceed a prespecified maximum value.

By contrast the **broadcast mode** of operation in which several distinct messages (and possibly a common message) are transmitted simultaneously from a single station to different receivers has not received sufficient attention. In the conference paper [15] we introduced broadcast direct-sequence (DS) spread-spectrum systems and presented some preliminary results. In this paper we examine several alternatives of broadcast spread-spectrum systems and evaluate their **broadcast capability**, defined as the maximum

number of **simultaneous distinct messages** that can be transmitted to distant receivers from a single station at a given received bit-error-rate. This quantity provides a useful measure of the capacity of hub-to-mobile or satellite-to-earth-station links in communication networks such as the MSAT or VSAT. When **bursty** data or voice traffic is dominant in such networks, the above forms of spread-spectrum code-division multiple-access (CDMA) provide a viable alternative to frequency-division (FDMA) or time-division (TDMA) multiple-access.

Two forms of spread-spectrum signaling: direct-sequence (DS) and hybrid direct-sequence/frequency-hopped (DS-FH) are shown to provide high broadcast capability especially when used in conjunction with forward-error-control coding schemes.

In DS broadcast systems the different messages are DS modulated employing distinct signature sequences and either transmitted on distinct carriers, or grouped together in pairs and transmitted on distinct pairs of orthogonal carriers, or grouped together in two groups, added together in each group, and then transmitted on two orthogonal carriers. The receivers use matched filters and replicas of the different transmitted signature sequences. The network formed by the single transmitter and the receivers is termed a local area radio network (LARN) and is assumed to be free of the near-far problem for this application where only DS/SS signaling is used. Systems with chip-synchronous signals and systems where random delays are introduced between the signals are considered. The average error probability of all systems is evaluated using the characteristic-function and Gaussian-approximation techniques.

Hybrid (DS-FH) broadcast systems are similar to the DS broadcast systems except that now the sum of the DS spread signals (multiplexed in any of the three methods mentioned in the previous paragraph) is frequency-hopped according to a hopping pattern instead of a single carrier frequency. The network topology consists now of several LARN's. Each distinct LARN is assigned a distinct hopping pattern. All the stations in a LARN employ the same hopping pattern and distinct signature sequences. The signature sequences used by distinct LARN's are also distinct. In this way the effect of the near-far problem of DS is reduced, because DS modulation is used alone only inside the LARN's where the distances between stations do not differ considerably, whereas for communica-

tion between different LARN's the use of FH together with DS modulation improves the performance.

This paper is organized as follows. In Sections II and III DS/SS and hybrid DS-FH/SS broadcast schemes are considered. In each section we first present the system models for both synchronous and asynchronous systems and the three multiplexing techniques mentioned above, and then derive the error probability at the output of the correlation receiver for phase-shift-keying (PSK) modulation with coherent demodulation, based on the characteristic-function and the Gaussian-approximation methods. Next, in Section IV the use of forward error-control coding in broadcast direct-sequence and hybrid spread-spectrum systems is investigated. In particular the performance of convolutional codes with Viterbi decoding and Reed-Solomon codes with bounded distance decoding is evaluated. Finally, in Section V a variety of numerical results illustrating the broadcast capability of uncoded and coded DS/SS and hybrid DS-FH/SS systems are presented and conclusions are drawn.

## II. DS/SS Broadcast Systems

### A. System Models

Three different ways of multiplexing the direct-sequence (DS) signals at the common transmitter are presented in the following. They employ only two orthogonal carriers (System 1), several distinct pairs of orthogonal carriers (System 2), and completely distinct carriers (System 3), respectively.

#### a) System 1.

Two orthogonal (in phase and quadrature) carrier signals of frequency  $f_c$  are used to transmit  $2K$  DS-modulated signals. The overall transmitted signal for the synchronous case is of the form:

$$S(t) = \left[ \sum_{k=1}^K \sqrt{2P} b_{2k-1}(t) a_{2k-1}(t) \Psi(t) \right] \sin(2\pi f_c t) + \left[ \sum_{k=1}^K \sqrt{2P} b_{2k}(t) a_{2k}(t) \Psi(t) \right] \cos(2\pi f_c t) \quad (1)$$

where  $P$  is the common signal power and  $b_i(t), i \in \{1, 2, \dots, 2K\}$  is the  $i$ th user data waveform consisting of a sequence of mutually independent rectangular pulses  $b_n^{(i)}$  of duration  $T$  and amplitude taking values  $+1$  or  $-1$  with equal probability.  $\Psi(t)$  is the shaping waveform as defined in [2].  $a_i(t), i \in \{1, 2, \dots, 2K\}$  is the code waveform for the  $i$ th user. This consists of the periodic sequence  $\{a_n^{(i)}\}$  of period  $N$ , (where  $a_n^{(i)}$  takes values  $+1$  or  $-1$  with equal probability) and the integer  $N = T/T_c$  is the number of chips per bit. For the asynchronous case  $S(t)$  has the form

$$S(t) = \left[ \sum_{k=1}^K \sqrt{2P} b_{2k-1}(t - \tau_{2k-1}) a_{2k-1}(t - \tau_{2k-1}) \Psi(t - \tau_{2k-1}) \right] \sin(2\pi f_c t) \\ + \left[ \sum_{k=1}^K \sqrt{2P} b_{2k}(t - \tau_{2k}) a_{2k}(t - \tau_{2k}) \Psi(t - \tau_{2k}) \right] \cos(2\pi f_c t) \quad (2)$$

where it is assumed that the delays ( $\tau_{2k-1}$  and  $\tau_{2k}; k \in \{1, 2, \dots, K\}$ ) are i.i.d and uniformly distributed in  $[0, T]$ . If the number of message signals to be transmitted is odd (say  $2K + 1$  instead of  $2K$ ), the first  $2K$  signals are multiplexed as in (2) and the  $2K + 1$ th signal is transmitted on the  $\sin(2\pi f_c t)$  carrier [i.e.,  $K$  is replaced by  $K + 1$  in the sum inside the first bracket in (2)].

The above delays  $\tau_{2k'-1,k}$ ,  $\tau_{2k',k}$ , and  $\tau_k$  may appear when, for example,  $S(t)$  is the transmitted signal from a relay which has gathered and retransmitted the  $2K$  incoming asynchronous DS signals. Another possibility is that the delays are purposefully introduced at the transmitter. This might be desirable, since, as shown in [18] for DS/SS multiple-access systems and in this paper (see also [15] for some preliminary results) for broadcast DS/SS systems employing random signature sequences, the performance of asynchronous systems is superior to that of synchronous systems. However, in this case the complexity of the transmitter increases.

The received signal at any receiver has the form

$$r(t) = S(t) + n(t) \quad (3)$$

where  $n(t)$  is a zero-mean Gaussian noise with spectral density  $N_0/2$ . The  $i$ th receiver



is simply a correlation receiver matched to the  $i$  th signal component of the composite transmitted signal of (2).

### b) System 2.

In this model the  $2K$  transmitted signals are combined together in  $K$  QPSK-type pairs as in the following expression

$$S(t) = \sum_{k=1}^K [b_{2k-1}(t)a_{2k-1}(t)\Psi(t)\sin(2\pi f_c t + \theta_k) + b_{2k}(t)a_{2k}(t)\Psi(t)\cos(2\pi f_c t + \theta_k)] \quad (4)$$

for the synchronous, and

$$S(t) = \sum_{k=1}^K [b_{2k-1}(t - \tau_{2k-1})a_{2k-1}(t - \tau_{2k-1})\Psi(t - \tau_{2k-1})\sin(2\pi f_c t + \theta_k) + b_{2k}(t - \tau_{2k})a_{2k}(t - \tau_{2k})\Psi(t - \tau_{2k})\cos(2\pi f_c t + \theta_k)] \quad (5)$$

for the asynchronous systems. Therefore  $K$  distinct pairs of orthogonal carriers (with the same frequency  $f_c$  but different phase angles) are used. All the phases are i.i.d and uniformly distributed in  $[0, 2\pi]$ . If the number of message signals is odd (i.e.,  $2K + 1$  instead of  $2K$ ) the first  $2K$  signals are multiplexed as in (4) and the  $2K + 1$  th signal is transmitted on the separate carrier  $\sin(2\pi f_c t + \theta_{2K+1})$ .

### c) System 3.

This model is the traditional binary multiple-access (MA) DS/SS system in which all  $2K$  transmitted signals are carried by distinct carriers as follows

$$S(t) = \sum_{k=1}^{2K} \sqrt{2P} b_k(t) a_k(t) \Psi(t) \cos(2\pi f_c t + \theta_k) \quad (6)$$

For the asynchronous case,  $t$  is replaced by  $t - \tau_k$ . If the number of messages to be transmitted is odd we only need replace  $2K$  by  $2K + 1$  in (6).

## B. Average Error Probability

Since we are interested in the average performance, all usual assumptions (see [1]-[3], [5], and [18]) about random mutually independent data streams, phase angles, and delays (in the asynchronous case), which are uniformly distributed in the appropriate sets, are made. Although the analytical techniques of this paper (see [3], [18], and [15]) allow us to obtain results for deterministic signature sequences, we restrict ourselves to random signature sequences. This is a useful modeling assumption, because it makes all SS signals and receivers equivalent and facilitates the analysis and presentation of numerical results. It is also a practical modeling assumption, since the random models are quite satisfactory for long signature sequences about whose structure not much information is available. In particular for *random signature sequences* it is assumed that the values of each sequence in different chips are mutually independent random variables assuming the values  $+1$  or  $-1$  with equal probability. Distinct sequences are also mutually independent.

### a) System 1.

The output of the  $m$ th correlation receiver ( $m = 2i - 1$  or  $m = 2i$  and  $i \in \{1, 2, \dots, 2K\}$ ) is obtained as

$$Z_m = \eta_m + \sqrt{\frac{P}{2}} \cdot T \left\{ b_0^{(m)} + \sum_{k=1, k \neq i}^K [I_{2k-1, m} + I_{2k, m}] \right\} \quad (7)$$

where  $\eta_m$  is a zero-mean Gaussian random variable with variance  $\frac{N_0 T}{4} \cdot I_{2k-1, m}$  and  $I_{2k, m}$  are broadcast interferences and are zero for  $m = 2i$  and  $m = 2i - 1$ , respectively. Following the method described in [3], the average probability of error is

$$\bar{P}_e = Q(\alpha) + \frac{1}{\pi} \int_0^\infty \frac{\sin u}{u} \cdot \exp\left(\frac{-u^2}{2\alpha^2}\right) [1 - \bar{\Phi}(u)] du \quad (8)$$

where  $\alpha = \sqrt{\frac{2Eb}{N_0}}$  and  $\bar{\Phi}(u)$  is the average characteristic function defined as  $\bar{\Phi}(u) = E[\Phi_{2i-1}(u)] = E[\Phi_{2i}(u)]$ , where  $\Phi_m(u)$  is the characteristic function for the broadcast

interference at the output of the  $m$ th receiver. For the synchronous case  $I_{2k-1,2i-1}$  and  $I_{2k,2i}$  are

$$I_{2k-1,2i-1} = \frac{1}{N} b_1^{(2k-1)} \cdot \theta_{2k-1,2i-1}(0) \quad (9a)$$

$$I_{2k,2i} = \frac{1}{N} b_1^{(2k)} \cdot \theta_{2k,2i}(0) \quad (9b)$$

where  $\theta_{k,\ell}(\cdot)$  is the periodic (even) cross-correlation function [1] for the sequences  $\{a_n^{(k)}\}$  and  $\{a_n^{(\ell)}\}$ . According to the procedure suggested in [15] for computing the characteristic function of the other-user interference in DS/SS systems employing random signature sequences (see also [18]),  $\bar{\Phi}(u)$  is derived as

$$\bar{\Phi}(u) = \bar{E}_i \left\{ \prod_{k \neq i} E_{b_1^{(2k)}} \left[ \bar{E}_k \left\{ \exp(ju I_{2k,2i}) / \underline{a}^{(2i)}, b_1^{(2k)} \right\} \right] \right\} \quad (10)$$

where  $\bar{E}_k$  is the expectation with respect to the sequence  $\{a_n^{(k)}\}$  which takes values in  $\{-1, 1\}^N$  with equal probability.  $E_{b_1^{(2k)}}$  is the average over the values of  $b_1^{(2k)}$  in  $\{-1, 1\}$  and  $\bar{E}_i$  is the average over all the values of the sequence  $\{a_n^{(2i)}\}$  which are in  $\{-1, 1\}^N$ . Using (9b) and replacing  $\theta_{2k,2i}(0)$  by  $\sum_{j=0}^{N-1} a_j^{(2k)} \cdot a_j^{(2i)}$ ,

$$\bar{E}_k \left\{ \exp(ju I_{2k,2i}) / \underline{a}^{(2i)}, b_1^{(2k)} \right\} = \prod_{j=0}^{N-1} \cos \left( \frac{u}{N} b_1^{(2k)} \cdot a_j^{(2i)} \right) \quad (11)$$

Taking the expectation  $E_{b_1^{(2k)}}$  of (11) and interchanging  $\prod_{k \neq i} \prod_{j=0}^{N-1}$  with  $\prod_{j=0}^{N-1} \prod_{k \neq i}$ , we have

$$\prod_{k \neq i} E_{b_1^{(2k)}} [\bar{E}_k \{\cdot\}] = \prod_{j=0}^{N-1} \cos^{K-1} \left( \frac{u}{N} a_j^{(2i)} \right) \quad (12)$$

and finally taking the expectation  $\bar{E}_i$  of (12)

$$\bar{\Phi}(u) = \cos^{N(K-1)} \left( \frac{u}{N} \right) \quad (13)$$

This expression has also been derived in [5] for a multiple-access DS/SS system via a different approach.

The average error probability  $\overline{P}_e^G$  is obtained through the Gaussian approximation and defined as  $Q(\overline{SNR})$ , where  $\overline{SNR}$  is the average signal to noise ratio at the output of the correlation receiver

$$\overline{SNR} = \left[ \left( \frac{2E_b}{N_0} \right)^{-1} + \frac{K-1}{N} \right]^{-1/2} \quad (14)$$

In asynchronous systems, (7) still holds but the interference components are as follows

$$I_{2k-1,2i-1} = T^{-1} \left[ b_1^{(2k-1)} \cdot R_{2k-1,2i-1}(\tau_{2k-1}) + b_2^{(2k-1)} \cdot \hat{R}_{2k-1,2i-1}(\tau_{2k-1}) \right] \quad (15a)$$

$$I_{2k,2i} = T^{-1} \left[ b_1^{(2k)} \cdot R_{2k,2i}(\tau_{2k}) + b_2^{(2k)} \cdot \hat{R}_{2k,2i}(\tau_{2k}) \right] \quad (15b)$$

where  $R_{k,i}(\cdot)$  and  $\hat{R}_{k,i}(\cdot)$  are continuous partial cross-correlation functions as defined in [1]. Using the same method as described above for the synchronous case we derive for the asynchronous case (see [18] for the details of DS/SS multiple-access systems),  $\overline{P}_e$  is derived from (8) where  $\overline{\Phi}(u)$  is

$$\overline{\Phi}(u) = \left( \frac{1}{2} \right)^{N-1} \sum_{n=0}^{N-1} \binom{N-1}{n} [\Phi_{N,n}(u)]^{K-1} \quad (16)$$

where

$$\Phi_{N,n}(u) = \frac{2}{T_c} \int_0^{T_c/2} \tilde{f}_c(N, n, u; \tau) d\tau \quad (17)$$

and

$$\begin{aligned} \tilde{f}_c(N, n, u; \tau) = & \cos \left[ \frac{u}{T} R_\Psi(\tau) \right] \cos \left[ \frac{u}{T} \hat{R}_\Psi(\tau) \right] \cdot \\ & \cdot \cos^n \left\{ \frac{u}{T} \left[ R_\Psi(\tau) + \hat{R}_\Psi(\tau) \right] \right\} \cos^{N-1-n} \left\{ \frac{u}{T} \left[ R_\Psi(\tau) - \hat{R}_\Psi(\tau) \right] \right\} \end{aligned} \quad (18)$$

where  $R_\Psi(\tau)$  and  $\hat{R}_\Psi(\tau)$  are partial auto correlation functions for the chip waveforms and are defined as  $\hat{R}_\Psi(s) = \int_s^{T_c} \Psi(t)\Psi(t-s)dt$  and  $R_\Psi(s) = \hat{R}_\Psi(T_c - s)$  for  $0 \leq s \leq T_c$ . For a rectangular waveform chip,  $\Phi_{N,n}(u)$  can be more simplified as follows

If  $N - n - 1$  is even, let  $m \triangleq (N - n - 1)/2$ , then

$$\begin{aligned} \Phi_{N,n}(u) &= \left(\frac{1}{2}\right)^{2m+1} \cdot \frac{u}{N} \cos^n\left(\frac{u}{N}\right) \cdot \\ &\cdot \left\{ \sum_{k=1}^m \binom{2m}{m-k} \left[ \left( \frac{1}{2k} + \frac{1}{2k+1} \right) \sin[(2k+1)u/N] + \left( \frac{1}{2k} + \frac{1}{2k-1} \right) \sin[(2k-1)u/N] \right] \right. \\ &\left. + \binom{2m}{m} \left[ \sin\left(\frac{u}{N}\right) + \frac{u}{N} \cos\left(\frac{u}{N}\right) \right] \right\} \end{aligned} \quad (19a)$$

If  $N - n - 1$  is odd, let  $m \triangleq (N - n - 2)/2$ , then

$$\begin{aligned} \Phi_{N,n}(u) &= \left(\frac{1}{2}\right)^{2m+2} \cdot \frac{u}{N} \cos^n\left(\frac{u}{N}\right) \cdot \\ &\cdot \left\{ \sum_{k=1}^m \binom{2m}{m-k} \left[ \left( \frac{1}{2k+1} + \frac{(2m+1)(m+1)}{(k+1)(m+k+1)(m+k+2)} \right) \sin\left(2(k+1)\frac{u}{N}\right) \right. \right. \\ &\quad \left. \left. + \left( \frac{1}{2k+1} + \frac{1}{2k-1} \right) \sin\left(2k\frac{u}{N}\right) + \frac{1}{2k-1} \cdot \sin\left(2(k-1)\frac{u}{N}\right) \right] \right. \\ &\left. + \binom{2m}{m} \cdot \frac{3(m+1)}{m+2} \sin\left(2\frac{u}{N}\right) + \binom{2m}{m} \frac{2m+1}{m+1} \cdot \frac{u}{N} \right\} \end{aligned} \quad (19b)$$

Similarly  $\overline{P}_e^G$  is computed as  $Q(\overline{SNR})$  where

$$\overline{SNR} = \left[ \left( \frac{2E_b}{N_0} \right)^{-1} + \frac{2(K-1)}{N} m_\Psi \right]^{-1/2} \quad (20)$$

and  $m_\Psi = T_c^{-3} \int_0^{T_c} R_\Psi^2(\tau) d\tau = T_c^{-3} \int_0^{T_c} \hat{R}_\Psi^2(\tau) d\tau$ .

## b) System 2.

The average error probability for this system turns out to be identical to that of the quaternary DS/SSMA system without offset which has already been evaluated in [18].  $\bar{P}_e$  for both the synchronous and asynchronous systems is computed from (8). For the synchronous systems

$$\bar{\Phi}(u) = \left[ \frac{2}{\pi} \int_0^{\pi/2} \cos^N \left( \frac{u}{N} \cos \theta \right) \cos^N \left( \frac{u}{N} \sin \theta \right) d\theta \right]^{K-1} \quad (21)$$

For the asynchronous case (16) holds, where  $\Phi_{N,n}(u)$  is replaced by  $\tilde{\Phi}_{N,n}(u)$  as

$$\tilde{\Phi}_{N,n}(u) = \frac{4}{\pi T_c} \int_0^{\pi/2} \int_0^{T_c/2} f_c(N, n, u; \tau, \theta) f_s(N, n, u; \tau, \theta) d\tau d\theta \quad (22)$$

and from (18)

$$f_c(N, n, u; \tau, \theta) = \tilde{f}_c(N, n, u \cos \theta; \tau) \quad (23a)$$

$$f_s(N, n, u; \tau, \theta) = \tilde{f}_c(N, n, u \sin \theta; \tau) \quad (23b)$$

Gaussian approximations for the error probability are exactly the same as the ones in System 1. Therefore, (14) and (20) hold for this system as well.

## c) System 3.

The performance of this broadcast DS/SS system which is equivalent to a multiple-access DS/SS system is already been evaluated (see [18]). Therefore, it is not repeated here.

## III. Hybrid Direct-Sequence/Frequency-Hopped Systems

### A. System Models

Hybrid (DS-FH) broadcast systems are similar to the DS broadcast systems except that now the sum of the DS spread signals (multiplexed in any of the three methods described

as Systems 1, 2, and 3 in Section II.A) is frequency-hopped according to a hopping pattern instead of a single carrier frequency. Refer to the discussion in the introduction about the concept of the local area radio network (LARN). We again distinguish the broadcast hybrid (DS-FH/SS) systems in three types as we did for the DS/SS broadcast systems.

#### a) System 1.

In this system model there are  $2K_d$  users in each local area radio network (LARN), which are direct-sequence modulated as in System 1 of Section II and share the same hopping pattern.  $K_h$  is the number of distinct LARN's or frequency hopping patterns in the network. Therefore,  $K = 2K_d K_h$  is the total number of distinct signals and receivers.

For synchronous systems, the transmitted signal has the form

$$S(t) = \sum_{k=1}^{K_h} \left\{ \left[ \sum_{k'=1}^{K_d} \sqrt{2P} b_{2k'-1,k}(t) a_{2k'-1,k}(t) \Psi(t) \right] \sin \{2\pi [f_c + f_k(t)] t + \alpha_k(t)\} \right. \\ \left. + \left[ \sum_{k'=1}^{K_d} \sqrt{2P} b_{2k',k}(t) a_{2k',k}(t) \Psi(t) \right] \cos \{2\pi [f_c + f_k(t)] t + \alpha_k(t)\} \right\} \quad (24)$$

where  $b_{k',k}(t)$  and  $a_{k',k}(t)$  are the data and signature waveforms for  $1 \leq k' \leq 2K_d$  and  $1 \leq k \leq K_h$ .  $f_k(t)$  is the frequency-hopping waveform, which is a sequence of rectangular pulses of duration  $T_h$  (dwell time) associated with a hopping pattern  $\{f_j^{(k)}\}$  taking values in a set of  $q$  available frequencies (see [7]). There are  $N_b$  data bits transmitted during each dwell time.  $\alpha_k(t)$  is the phase waveform introduced by the local oscillator which generates  $f_k(t)$ . At the  $(i', i)$ th receiver, the received signal  $r(t)$  is first passed through a frequency dehopper which is matched to the  $i$ th frequency pattern and then is coherently demodulated. The outputs of the  $(2i' - 1)$ th and  $(2i')$ th matched filter that follow the  $i$ th dehopper are

$$Z_{2i'-1,i} = \int_0^T r_{di}(t) \Psi(t) a_{2i'-1,i}(t) \sin(2\pi f_c t) dt \quad (25a)$$

$$Z_{2i',i} = \int_0^T r_{di}(t) \Psi(t) a_{2i',i}(t) \cos(2\pi f_c t) dt \quad (25b)$$

for  $1 \leq i' \leq K_d$  and  $1 \leq i \leq K_h$ .

In (25)  $r_{di}(t)$  is the signal at the output of the dehopper, that is, the output of an appropriate band-pass filter to an input  $r(t) \cos [2\pi f_i(t) + \beta_i(t)]$ , where  $\beta_i(t)$  is the phase waveform introduced by the local oscillator generating  $f_i(t)$  (see [7], [8], [9] ) and the received signal  $r(t)$  is given by (3).

In the asynchronous mode of operation, the broadcast signal is still given by (24), provided that we replace  $t$  by  $t - \tau_{2k'-1,k}$  and  $t - \tau_{2k',k}$  in the arguments of the data waveforms, the signature sequences, and the shaping waveforms, and  $t$  by  $t - \tau_k$  in the arguments of the frequency-hopping waveforms and the phase waveforms. Finally, if the number of message signals to be transmitted to the receivers of a particular LARN is odd ( $2K_d + 1$  instead of  $2K_d$ ) we follow the same method as in the corresponding case (System 1) of the DS/SS system to take care of the transmission of the extra signal.

#### b) System 2.

In this model the  $2K_d$  transmitted signals in a LARN are direct- sequence modulated as in the model described for System 2 in Section II. The overall transmitted signal for the synchronous mode takes the form

$$S(t) = \sum_{k=1}^{K_h} \sum_{k'=1}^{K_d} \left[ \sqrt{2P} b_{2k'-1,k}(t) a_{2k'-1,k}(t) \Psi(t) \sin \{2\pi [f_c + f_k(t)] t + \alpha_k(t) + \theta_{k',k}\} \right. \\ \left. + \sqrt{2P} b_{2k',k}(t) a_{2k',k}(t) \Psi(t) \cos \{2\pi [f_c + f_k(t)] t + \alpha_k(t) + \theta_{k',k}\} \right] \quad (26)$$

The received signal  $r(t)$  is demodulated in a manner similar to that of System 1, and (25) still holds. For the asynchronous mode of operation the broadcast signal is still given by (26), provided that  $t$  is replaced by  $t - \tau_{2k'-1,k}$  and  $t - \tau_{2k',k}$  in the arguments of the data waveforms, the signature sequences, and the shaping waveforms, and  $t$  by  $t - \tau_k$  in the arguments of the frequency-hopping waveforms and the phase waveforms. Finally, for the case that the number of message signals in a LARN is odd ( $2K_d + 1$  instead of  $2K_d$ ) we use the same method as for the corresponding case of System 2 of Section II to take care of the transmission of the extra signal.



### c) System 3.

For this model, in each LARN, signals are direct-sequence modulated in the same fashion as System 3 in section II. For the synchronous case,  $S(t)$  is

$$S(t) = \sum_{k=1}^{K_h} \sum_{k'=1}^{K_d} \sqrt{2P} b_{k',k}(t) a_{k',k}(t) \Psi(t) \cos \{2\pi [f_c + f_k(t)] t + \alpha_k(t) + \theta_{k',k}\} \quad (27)$$

Notice that the number of receivers in each LARN is  $K_d$ . Hence, the total number of receivers in the network is  $K = K_h K_d$ . For the asynchronous case,  $t$  is replaced by  $t - \tau_{k,k'}$  and  $t - \tau_k$  as described for the other systems.

## B. Average Error Probability

To examine the performance of broadcast hybrid SS systems we need to evaluate the average error probability (or bit error rate BER) and from that the broadcast capability. We make the same modeling assumptions for the data streams, phase angles, and time delays that were made in [7]-[9] for frequency-hopped and hybrid systems. In this case, besides the signature sequences being modeled as random, the frequency-hopping patterns are also modeled as *random memoryless hopping patterns* (see [7]), that is, it is assumed that during any dwell time each frequency-hopper visits all  $q$  available frequencies with equal probability and that frequencies visited during different dwell times are mutually independent. Again, distinct hopping patterns are mutually independent. This random model is used for the same reasons as the random-signatures model of DS/SS systems—refer to the corresponding discussion at the beginning of Section II.B for broadcast DS/SS systems. In particular this model can be quite accurate for long frequency-hopping patterns or for hopping patterns about whose structure there is not sufficient information.

### a) System 1.

We consider coherent demodulation for this hybrid SS broadcast system. During the reception of the  $(2i' - 1)$ th message ( $1 \leq i' \leq K_d$ ) in the  $i$ th LARN ( $1 \leq i \leq K_h$ ), the interference term is

$$I_{2i'-1,i} = \sum_{k'=1, k' \neq i'}^{K_d} (I_{2k'-1,i;2i'-1,i}) + \sum_{k=1, k \neq i}^{K_h} \sum_{k'=1}^{K_d} (I_{2k'-1,k;2i'-1,i} + I_{2k',k;2i'-1,i}) \quad (28)$$

In the synchronous system, these terms takes the values

$$I_{2k'-1,i;2i'-1,i} = b_0^{(2k'-1,i)} \theta_{2k'-1,i;2i'-1,i}(0)/N \quad (29a)$$

$$I_{2k'-1,k;2i'-1,i} = \begin{cases} b_0^{(2k'-1,k)} [\theta_{2k'-1,k;2i'-1,i}(0)/N] \cos \Psi_k, & w.p. \quad P \\ 0 & w.p. \quad 1 - P \end{cases} \quad (29b)$$

$$I_{2k',k;2i'-1,i} = \begin{cases} b_0^{(2k',k)} [\theta_{2k',k;2i'-1,i}(0)/N] (-\sin \Psi_k), & w.p. \quad P \\ 0 & w.p. \quad 1 - P \end{cases} \quad (29c)$$

In (29b) and (29c),  $\Psi_k$  is a phase angle uniformly distributed in  $[0, 2\pi]$ , *w.p.* is an abbreviation for the expression: “with probability”, and  $P = 1/q$  is the probability of a hit. Notice that similar expressions as (28) and (29) can be written for  $(2i')$ th message in the  $i$ th LARN.

Following a procedure similar to the one used in DS systems in Section II and in [18], the characteristic function of  $I_{2i'-1,i}$  (or  $I_{2i',i}$ ), conditioned on having  $J$  hits ( $0 \leq J \leq K_h - 1$ ), turns out to be

$$\Phi(J; u) = \left[ \cos \left( \frac{u}{N} \right) \right]^{N(K_d-1)} \cdot \left\{ \frac{2}{\pi} \int_0^{\pi/2} \left[ \cos \left( \frac{u}{N} \cos \theta \right) \cos \left( \frac{u}{N} \sin \theta \right) \right]^{NK_d} d\theta \right\}^J, \quad (30)$$

and the average probability of error  $\bar{P}_e$  is given by

$$\bar{P}_e = \sum_{J=0}^{K_h-1} \binom{K_h-1}{J} P^J (1-P)^{K_h-1-J} P_{e,f}(J) \quad (31)$$

where  $P_{e,f}(J)$  is the conditional error probability given that  $J$  hits ( $0 \leq J \leq K_h - 1$ ) have occurred and is computed from (8) by replacing  $\bar{\Phi}(u)$  with  $\Phi(J, u)$ . Combining this equation with (31) and after some further simplifications, we have that  $\bar{P}_e$  is given by (8) where  $\bar{\Phi}(u)$  is

$$\bar{\Phi}(u) = \left[ \cos \left( \frac{u}{N} \right) \right]^{N(K_d-1)} \cdot \left\{ 1 - P + P \cdot \frac{2}{\pi} \int_0^{\pi/2} \left[ \cos \left( \frac{u}{N} \cos \theta \right) \cos \left( \frac{u}{N} \sin \theta \right) \right]^{NK_d} d\theta \right\}^{K_h-1} \quad (32)$$

The average error probability via Gaussian approximation  $\bar{P}_e^G$  is given by (31) where  $P_{e,f}(J)$  is replaced by

$$P_{e,f}(J) = Q(\overline{SNR}) = Q\left\{\left[\alpha^{-1} + (JK_d + K_d - 1)/N\right]^{-1/2}\right\} \quad (33)$$

In the asynchronous mode of operation and under the assumption that all hits are full hits (see [8]),  $\bar{P}_e$  is upper bounded by (8). The interference term  $I_{2i'-1,i}$  is still given by (28), but (29) should be replaced by expressions which involve the continuous-time partial crosscorrelation functions  $R_{\ell,j}(\cdot)$  and  $\hat{R}_{\ell,j}(\cdot)$ ,  $\tau_{2k'-1,k}$ ,  $\tau_{2k',k}$ , and  $\tau_k$ .  $\bar{\Phi}(u)$  is evaluated in the same way as the synchronous case to be

$$\bar{\Phi}(u) = \left(\frac{1}{2}\right)^{N-1} \sum_{n=0}^{N-1} \binom{N-1}{n} [\Phi_{N,n}(u)]^{K_d-1} \cdot \left[1 - P + P\hat{\Phi}_{N,n}(u)\right]^{K_h-1} \quad (34)$$

where  $\Phi_{N,n}(u)$  is given in (17),

$$\hat{\Phi}_{N,n}(u) = \frac{2}{\pi} \int_0^{\pi/2} \left[ \frac{2}{T_c} \int_0^{T_c/2} f_c(N, n, u; \tau, \theta) f_s(N, n, u; \tau, \theta) d\tau \right]^{K_d} d\theta \quad (35)$$

and  $f_c(N, n, u; \tau, \theta)$  and  $f_s(N, n, u; \tau, \theta)$  are given in (23a) and (23b). Notice that the probability of a hit for asynchronous systems is  $P = \left[1 + \frac{1}{N_b} \left(1 - \frac{1}{q}\right)\right] \frac{1}{q}$  for random memoryless hopping patterns [7].

The Gaussian error probability  $\bar{P}_e^G$  is upper bounded by (31), where  $P_{e,f}(J)$  is replaced by

$$P_{e,f}(J) = Q(\overline{SNR}) = Q\left(\left[\alpha^{-1} + 2(JK_d + K_d - 1) \frac{m_\Psi}{N}\right]^{-1/2}\right) \quad (36)$$

## b) System 2.

At the output of the  $(2i' - 1)$ th correlation receiver ( $1 \leq i' \leq K_d$ ) in the  $i$ th LARN ( $1 \leq i \leq K_h$ ), the interference term  $I_{2i'-1,i}$  is given by (28) with  $I_{2k',i;2i'-1,i}$  as the extra term in the first summation. In the synchronous case

$$I_{2k',i;2i'-1,i} = \left[ b_o^{(2k',i)} \cdot \theta_{2k',i;2i'-1,i}(0)/N \right] (-\sin \theta_{k',i}) \quad (37)$$

(29a) still holds if we multiply the right hand side by  $\cos \theta_{k',i}$ . (29b) and (29c) also remain valid if  $\Psi_k$  is replaced by  $\theta_{k',k}$ .

Similarly,  $\overline{P}_e$  is given by (8) and  $\overline{\Phi}(u)$  is

$$\overline{\Phi}(u) = [\Phi_1(u)]^{K_d-1} \cdot \left\{ 1 - P + P [\Phi_1(u)]^{K_d} \right\}^{K_h-1} \quad (38)$$

where

$$\Phi_1(u) = \frac{2}{\pi} \int_0^{\pi/2} \cos^N \left( \frac{u}{N} \cos \theta \right) \cos^N \left( \frac{u}{N} \sin \theta \right) d\theta \quad (39)$$

and  $P = 1/q$ . The Gaussian error probability  $\overline{P}_e^G$  for the synchronous system is the same as the one for System 1.

In the asynchronous case,  $\overline{P}_e$  is upper bounded by (8) and  $\overline{\Phi}(u)$  turns out to be

$$\overline{\Phi}(u) = \left( \frac{1}{2} \right)^{N-1} \sum_{n=0}^{N-1} \binom{N-1}{n} \left[ \tilde{\Phi}_{N,n}(u) \right]^{K_d-1} \left\{ 1 - P + P \left[ \tilde{\Phi}_{N,n}(u) \right]^{K_d} \right\}^{K_h-1} \quad (40)$$

with  $\tilde{\Phi}_{N,n}(u)$  given in (22).  $P$  has the value  $(1 + 1/N_b)/q$ . The Gaussian error probability for the asynchronous system is the same as the one in the System 1.

### c) System 3.

The expressions derived for System 2 are valid here except that  $\Phi_1(u)$  in (38) should be replaced by

$$\Phi_1(u) = \frac{2}{\pi} \int_0^{\pi/2} \cos^N \left( \frac{u}{N} \cos \theta \right) d\theta \quad (41)$$

and  $\tilde{\Phi}_{N,n}(u)$  in (40) by  $\tilde{\Phi}_{N,n}(u)$  in (22) in which  $f_s(N, n, u; \tau, \theta)$  is omitted. For the Gaussian expressions, (33) and (36) are also valid if  $N$  is replaced by  $2N$ .

#### IV. PERFORMANCE OF CODED SYSTEMS

First we consider binary convolutional codes with Viterbi decoding. Interleaving at depth  $N_b$  for each dwell time is required. The bit error probability of the coded system is upperbounded by[17]

$$P_{b,c} \leq \sum_{j=d_{min}}^{\infty} w_j p_j \quad (42)$$

where

$$p_j = \begin{cases} \sum_{l=(j+1)/2}^j \binom{j}{l} \bar{P}_e^l (1 - \bar{P}_e)^{j-l} & j \text{ odd} \\ \frac{1}{2} \binom{j}{j/2} \bar{P}_e^{j/2} (1 - \bar{P}_e)^{j/2} + \sum_{l=j/2+1}^j \binom{j}{l} \bar{P}_e^l (1 - \bar{P}_e)^{j-l} & j \text{ even} \end{cases} \quad (43)$$

$p_j$  is the probability of the error event that the decoder chooses a path at distance  $j$  from the correct path.  $w_j$  is the total information weight of all sequences which produce path of distance  $j$ , and  $d_{min}$  is the free distance of the code. The values of  $\bar{P}_e$  are the uncoded error probability which have been obtained in the previous sections of this paper.

Then we consider the performance of any extended  $(n, k)$  *Reed-Solomon* code with  $n = 2^m$  ( $n$  is the codeword length and  $k$  is the number of information symbols per codeword). When BPSK modulation with coherent demodulation is employed, each code symbol contains  $m$  bits (the code alphabet is  $GF(2^m)$ ). In this case the hopping is slow (i.e.,  $N_b \gg 1$ ) and interleaving at depth  $N_b/m$  is required within each dwell time to guarantee the independence of the errors on the code symbols. When bounded distance decoding is employed, the probability of a decoder error for each symbol is given by[16]

$$P_{s,c} = \sum_{j=t+1}^n \frac{j}{n} \binom{n}{j} P_{s,un}^j (1 - P_{s,un})^{n-j} \quad (44)$$

where  $t = \lfloor (n - k)/2 \rfloor$  denotes the error correcting capability of the  $(n, k)$  RS code and  $P_{s,un}$  is the probability of a symbol error for the uncoded system. We can approximate  $P_{s,un}$  by the expression

$$P_{s,un} = 1 - (1 - \bar{P}_e)^m \quad (45)$$

where  $\overline{P}_e$  is the corresponding error probability from previous sections.

## V. NUMERICAL RESULTS AND CONCLUSIONS

In this section, we consider the numerical results for various types of systems described before. Numerical results for two classes of systems, direct sequence and hybrid (direct sequence/frequency hopped), are shown. In each case, synchronous and asynchronous systems are considered. the bit error probability obtained through the characteristic function method (for random signature sequences), is denoted by  $\overline{P}_e$ . The error probability obtained via the Gaussian approximation is denoted by  $\overline{P}_e^G$ . The performance of these systems in connection with two forward error control coding schemes is investigated.

Table 1 contains numerical results for  $\overline{P}_e$  and  $\overline{P}_e^G$  for all three broadcast direct sequence (DS) systems. Systems 1 and 2 perform almost identically for all cases (synchronous and asynchronous). Gaussian error probability is also a good approximation for these two systems, especially for larger values of  $N$ . But, it is rather optimistic for System 3. The asynchronous systems are superior to the corresponding synchronous ones in all cases. Figure 1 shows the broadcast capability of all three systems for  $N = 127$  and  $E_b/N_0 = 12db$ . Systems 1 and 2 perform better than System 3.

Table 2 contains the bit error probability for the broadcast hybrid systems. The results are demonstrated in parts a, b, c and d for different values of parameters. Synchronous System 1 and synchronous System 2 are almost identical and are better than synchronous System 3. But, asynchronous System 1 is better than asynchronous System 2, and both are better than asynchronous System 3. The Gaussian approximation to the error probability is a good approximation for all cases, except for asynchronous System 1, which is rather conservative and asynchronous System 3, which is rather optimistic. Figure 2 illustrates the broadcast capability of all these systems (except asynchronous System 1) for  $K_h = 10, N = 127, q = 100, N_b = 100, E_b/N_0 = 12db$ . Once again, it is observed that the asynchronous schemes exhibit better performance.

Finally, the performance of all direct sequence and hybrid systems using two types of coding schemes is demonstrated. Table 3 contains the numerical results for convolutional codes of rate 1/2 and the constraint length of 7 with Viterbi decoding for direct sequence

and hybrid systems. Comparison of these results with their counterparts in table 1b and table 2d, for uncoded systems (considering the same bandwidth expansion), shows considerable improvements for all systems using coding. The second coding scheme which was studied is Reed Solomon(RS) codes with bounded distance decoding. Tables 1b, 2d and 4 demonstrate the better performance for coded systems over uncoded ones. Figures 3 and 4 present the broadcast capability of one of the direct sequence systems and one of the hybrid systems for uncoded, convolutional coding and Reed Solomon coding, respectively. Convolutional coding shows a superior performance to that of the Reed Solomon coding for the cases considered.

In conclusion, in this paper we introduced a number of different DS/SS and hybrid (DS-FH/SS) system configurations and established that they demonstrate a considerable broadcast capability for transmitting distinct messages simultaneously from a common transmitter to several receivers. This broadcast capability is improved through the use of forward error-control coding.

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**Table 1. Error Probability for Broadcast DS/SS Systems**

(a) ( $2K = 6, N = 31$ )

|           | DS- System 1 |               |                      |             |               |                      |  | DS-System 2 |               |                      |             |               |                      |
|-----------|--------------|---------------|----------------------|-------------|---------------|----------------------|--|-------------|---------------|----------------------|-------------|---------------|----------------------|
|           | Sync         |               |                      | Async       |               |                      |  | Sync        |               |                      | Async       |               |                      |
| $E_b/N_0$ | $\bar{P}_e$  | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |  | $\bar{P}_e$ | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |
| 8         | .414         | .418          | ( $\times 10^{-2}$ ) | .215        | .212          | ( $\times 10^{-2}$ ) |  | .415        | .418          | ( $\times 10^{-2}$ ) | .215        | .212          | ( $\times 10^{-2}$ ) |
| 10        | .153         | .156          | ( $\times 10^{-2}$ ) | .545        | .521          | ( $\times 10^{-3}$ ) |  | .154        | .156          | ( $\times 10^{-2}$ ) | .549        | .521          | ( $\times 10^{-3}$ ) |
| 12        | .595         | .627          | ( $\times 10^{-3}$ ) | .139        | .125          | ( $\times 10^{-3}$ ) |  | .603        | .627          | ( $\times 10^{-3}$ ) | .142        | .125          | ( $\times 10^{-3}$ ) |
| 14        | .264         | .289          | ( $\times 10^{-3}$ ) | .411        | .335          | ( $\times 10^{-4}$ ) |  | .270        | .289          | ( $\times 10^{-3}$ ) | .428        | .335          | ( $\times 10^{-4}$ ) |
| 16        | .138         | .158          | ( $\times 10^{-3}$ ) | .153        | .111          | ( $\times 10^{-4}$ ) |  | .143        | .158          | ( $\times 10^{-3}$ ) | .162        | .111          | ( $\times 10^{-4}$ ) |

|           | DS System 3 |               |                      |             |               |                      |
|-----------|-------------|---------------|----------------------|-------------|---------------|----------------------|
|           | Sync        |               |                      | Async       |               |                      |
| $E_b/N_0$ | $\bar{P}_e$ | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |
| 8         | .662        | .619          | ( $\times 10^{-2}$ ) | .333        | .305          | ( $\times 10^{-2}$ ) |
| 10        | .331        | .283          | ( $\times 10^{-2}$ ) | 1.21        | .953          | ( $\times 10^{-3}$ ) |
| 12        | .188        | .142          | ( $\times 10^{-2}$ ) | .507        | .309          | ( $\times 10^{-3}$ ) |
| 14        | 1.23        | .806          | ( $\times 10^{-3}$ ) | .260        | .115          | ( $\times 10^{-3}$ ) |
| 16        | .910        | .527          | ( $\times 10^{-3}$ ) | 1.62        | .516          | ( $\times 10^{-4}$ ) |

(b) ( $2K = 24, N = 127$ )

|           | DS System 1 |               |                      |             |               |                      |  | DS System 2 |               |                      |             |               |                      |
|-----------|-------------|---------------|----------------------|-------------|---------------|----------------------|--|-------------|---------------|----------------------|-------------|---------------|----------------------|
|           | Sync        |               |                      | Async       |               |                      |  | Sync        |               |                      | Async       |               |                      |
| $E_b/N_0$ | $\bar{P}_e$ | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |  | $\bar{P}_e$ | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |
| 8         | .703        | .704          | ( $\times 10^{-2}$ ) | .346        | .345          | ( $\times 10^{-2}$ ) |  | .703        | .704          | ( $\times 10^{-2}$ ) | .346        | .345          | ( $\times 10^{-2}$ ) |
| 10        | .341        | .341          | ( $\times 10^{-2}$ ) | .117        | .116          | ( $\times 10^{-2}$ ) |  | .341        | .341          | ( $\times 10^{-2}$ ) | .117        | .116          | ( $\times 10^{-2}$ ) |
| 12        | .181        | .181          | ( $\times 10^{-2}$ ) | .421        | .409          | ( $\times 10^{-3}$ ) |  | .181        | .181          | ( $\times 10^{-2}$ ) | .422        | .409          | ( $\times 10^{-3}$ ) |
| 14        | .109        | .109          | ( $\times 10^{-2}$ ) | .175        | .166          | ( $\times 10^{-3}$ ) |  | .109        | .109          | ( $\times 10^{-2}$ ) | .176        | .166          | ( $\times 10^{-3}$ ) |
| 16        | .745        | .748          | ( $\times 10^{-3}$ ) | .878        | .811          | ( $\times 10^{-4}$ ) |  | .746        | .748          | ( $\times 10^{-3}$ ) | .881        | .811          | ( $\times 10^{-4}$ ) |

|           | DS System 3      |                    |                      |                  |                    |                      |
|-----------|------------------|--------------------|----------------------|------------------|--------------------|----------------------|
|           | Sync             |                    |                      | Async            |                    |                      |
| $E_b/N_0$ | $\overline{P}_e$ | $\overline{P}_e^G$ |                      | $\overline{P}_e$ | $\overline{P}_e^G$ |                      |
| 8         | .773             | .762               | ( $\times 10^{-2}$ ) | .380             | .372               | ( $\times 10^{-2}$ ) |
| 10        | .396             | .382               | ( $\times 10^{-2}$ ) | .138             | .131               | ( $\times 10^{-2}$ ) |
| 12        | .225             | .211               | ( $\times 10^{-2}$ ) | .550             | .486               | ( $\times 10^{-3}$ ) |
| 14        | .145             | .131               | ( $\times 10^{-2}$ ) | .257             | .208               | ( $\times 10^{-3}$ ) |
| 16        | 1.05             | .922               | ( $\times 10^{-3}$ ) | .145             | .107               | ( $\times 10^{-3}$ ) |

**Table 2. Error Probability for Broadcast Hybrid SS Systems**

(a) ( $K_h = 5$ ,  $2K_d = 6$ ,  $N = 31$ ,  $q = 50$ ,  $N_b = 100$ )

| $E_b/N_0$ | Hybrid System 1 |               |                      |             |               |                      | Hybrid System 2 |               |                      |             |               |                      |
|-----------|-----------------|---------------|----------------------|-------------|---------------|----------------------|-----------------|---------------|----------------------|-------------|---------------|----------------------|
|           | Sync            |               |                      | Async       |               |                      | Sync            |               |                      | Async       |               |                      |
|           | $\bar{P}_e$     | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      | $\bar{P}_e$     | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |
| 8         | .548            | .551          | ( $\times 10^{-2}$ ) | .245        | .279          | ( $\times 10^{-2}$ ) | .549            | .551          | ( $\times 10^{-2}$ ) | .283        | .279          | ( $\times 10^{-2}$ ) |
| 10        | .260            | .264          | ( $\times 10^{-2}$ ) | .722        | .967          | ( $\times 10^{-3}$ ) | .261            | .264          | ( $\times 10^{-2}$ ) | 1.00        | .967          | ( $\times 10^{-3}$ ) |
| 12        | .148            | .151          | ( $\times 10^{-2}$ ) | .245        | .426          | ( $\times 10^{-3}$ ) | .148            | .151          | ( $\times 10^{-2}$ ) | .438        | .412          | ( $\times 10^{-3}$ ) |
| 14        | .102            | .104          | ( $\times 10^{-2}$ ) | .109        | .251          | ( $\times 10^{-3}$ ) | .102            | .104          | ( $\times 10^{-2}$ ) | .270        | .251          | ( $\times 10^{-3}$ ) |
| 16        | .807            | .830          | ( $\times 10^{-3}$ ) | .644        | 1.83          | ( $\times 10^{-4}$ ) | .812            | .830          | ( $\times 10^{-3}$ ) | .198        | .183          | ( $\times 10^{-3}$ ) |

| $E_b/N_0$ | Hybrid System 3 |               |                      |             |               |                      |
|-----------|-----------------|---------------|----------------------|-------------|---------------|----------------------|
|           | Sync            |               |                      | Async       |               |                      |
|           | $\bar{P}_e$     | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |
| 8         | .805            | .764          | ( $\times 10^{-2}$ ) | .408        | .380          | ( $\times 10^{-2}$ ) |
| 10        | .454            | .406          | ( $\times 10^{-2}$ ) | .176        | .149          | ( $\times 10^{-2}$ ) |
| 12        | .294            | .247          | ( $\times 10^{-2}$ ) | .914        | .693          | ( $\times 10^{-3}$ ) |
| 14        | .218            | .174          | ( $\times 10^{-2}$ ) | .583        | .409          | ( $\times 10^{-3}$ ) |
| 16        | .179            | .138          | ( $\times 10^{-2}$ ) | .436        | .293          | ( $\times 10^{-3}$ ) |

(b) ( $K_h = 10$ ,  $2K_d = 6$ ,  $N = 31$ ,  $q = 100$ ,  $N_b = 100$ )

| $E_b/N_0$ | Hybrid System 1 |               |                      |             |               |                      | Hybrid System 2 |               |                      |             |               |                      |
|-----------|-----------------|---------------|----------------------|-------------|---------------|----------------------|-----------------|---------------|----------------------|-------------|---------------|----------------------|
|           | Sync            |               |                      | Async       |               |                      | Sync            |               |                      | Async       |               |                      |
|           | $\bar{P}_e$     | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      | $\bar{P}_e$     | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |
| 8         | .565            | .568          | ( $\times 10^{-2}$ ) | .249        | .288          | ( $\times 10^{-2}$ ) | .566            | .568          | ( $\times 10^{-2}$ ) | .292        | .288          | ( $\times 10^{-2}$ ) |
| 10        | .274            | .278          | ( $\times 10^{-2}$ ) | .746        | 1.03          | ( $\times 10^{-3}$ ) | .275            | .278          | ( $\times 10^{-2}$ ) | .106        | .103          | ( $\times 10^{-2}$ ) |
| 12        | .160            | .163          | ( $\times 10^{-2}$ ) | .259        | .469          | ( $\times 10^{-3}$ ) | .160            | .163          | ( $\times 10^{-3}$ ) | .496        | .469          | ( $\times 10^{-3}$ ) |
| 14        | .112            | .115          | ( $\times 10^{-2}$ ) | .119        | .284          | ( $\times 10^{-3}$ ) | .112            | .115          | ( $\times 10^{-2}$ ) | .304        | .284          | ( $\times 10^{-3}$ ) |
| 16        | .900            | .924          | ( $\times 10^{-3}$ ) | .718        | 2.10          | ( $\times 10^{-4}$ ) | .905            | .924          | ( $\times 10^{-3}$ ) | .226        | .210          | ( $\times 10^{-3}$ ) |

|           | Hybrid System 3 |               |                      |             |               |                      |
|-----------|-----------------|---------------|----------------------|-------------|---------------|----------------------|
|           | Sync            |               |                      | Async       |               |                      |
| $E_b/N_0$ | $\bar{P}_e$     | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |
| 8         | .824            | .783          | ( $\times 10^{-2}$ ) | .418        | .390          | ( $\times 10^{-2}$ ) |
| 10        | .469            | .422          | ( $\times 10^{-2}$ ) | .183        | .156          | ( $\times 10^{-2}$ ) |
| 12        | .308            | .261          | ( $\times 10^{-2}$ ) | .971        | .747          | ( $\times 10^{-3}$ ) |
| 14        | .230            | .186          | ( $\times 10^{-2}$ ) | .630        | .451          | ( $\times 10^{-3}$ ) |
| 16        | .190            | .149          | ( $\times 10^{-2}$ ) | .476        | .328          | ( $\times 10^{-3}$ ) |

(c) ( $K_h = 5$ ,  $2K_d = 24$ ,  $N = 127$ ,  $q = 50$ ,  $N_b = 100$ )

|           | Hybrid System 1 |               |                      |             |               |                      |  | Hybrid System 2 |               |                      |             |               |                      |
|-----------|-----------------|---------------|----------------------|-------------|---------------|----------------------|--|-----------------|---------------|----------------------|-------------|---------------|----------------------|
|           | Sync            |               |                      | Async       |               |                      |  | Sync            |               |                      | Async       |               |                      |
| $E_b/N_0$ | $\bar{P}_e$     | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |  | $\bar{P}_e$     | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |
| 8         | .848            | .848          | ( $\times 10^{-2}$ ) | .379        | .420          | ( $\times 10^{-2}$ ) |  | .848            | .848          | ( $\times 10^{-2}$ ) | .422        | .420          | ( $\times 10^{-2}$ ) |
| 10        | .465            | .465          | ( $\times 10^{-2}$ ) | .139        | .170          | ( $\times 10^{-2}$ ) |  | .465            | .465          | ( $\times 10^{-2}$ ) | .172        | .170          | ( $\times 10^{-2}$ ) |
| 12        | .289            | .289          | ( $\times 10^{-2}$ ) | .560        | .809          | ( $\times 10^{-3}$ ) |  | .289            | .289          | ( $\times 10^{-2}$ ) | .824        | .809          | ( $\times 10^{-3}$ ) |
| 14        | .205            | .205          | ( $\times 10^{-2}$ ) | .271        | .477          | ( $\times 10^{-3}$ ) |  | .205            | .205          | ( $\times 10^{-2}$ ) | .489        | .477          | ( $\times 10^{-3}$ ) |
| 16        | .163            | .163          | ( $\times 10^{-2}$ ) | .160        | .338          | ( $\times 10^{-3}$ ) |  | .163            | .163          | ( $\times 10^{-2}$ ) | .348        | .338          | ( $\times 10^{-3}$ ) |

|           | Hybrid System 3 |               |                      |             |               |                      |
|-----------|-----------------|---------------|----------------------|-------------|---------------|----------------------|
|           | Sync            |               |                      | Async       |               |                      |
| $E_b/N_0$ | $\bar{P}_e$     | $\bar{P}_e^G$ |                      | $\bar{P}_e$ | $\bar{P}_e^G$ |                      |
| 8         | .919            | .908          | ( $\times 10^{-2}$ ) | .457        | .449          | ( $\times 10^{-2}$ ) |
| 10        | .523            | .510          | ( $\times 10^{-2}$ ) | .195        | .187          | ( $\times 10^{-2}$ ) |
| 12        | .336            | .322          | ( $\times 10^{-2}$ ) | .978        | .907          | ( $\times 10^{-3}$ ) |
| 14        | .245            | .231          | ( $\times 10^{-2}$ ) | .597        | .538          | ( $\times 10^{-3}$ ) |
| 16        | .198            | .185          | ( $\times 10^{-2}$ ) | .431        | .382          | ( $\times 10^{-3}$ ) |

(d) ( $K_h = 10$ ,  $2K_d = 24$ ,  $N = 127$ ,  $q = 100$ ,  $N_b = 100$ )

|           | Hybrid System 1  |                    |                      |                  |                    |                      |  | Hybrid System 2  |                    |                      |  |                  |                    |                      |  |  |
|-----------|------------------|--------------------|----------------------|------------------|--------------------|----------------------|--|------------------|--------------------|----------------------|--|------------------|--------------------|----------------------|--|--|
|           | Sync             |                    |                      |                  | Async              |                      |  |                  | Sync               |                      |  |                  | Async              |                      |  |  |
| $E_b/N_0$ | $\overline{P}_e$ | $\overline{P}_e^G$ |                      | $\overline{P}_e$ | $\overline{P}_e^G$ |                      |  | $\overline{P}_e$ | $\overline{P}_e^G$ |                      |  | $\overline{P}_e$ | $\overline{P}_e^G$ |                      |  |  |
| 8         | .866             | .867               | ( $\times 10^{-2}$ ) | .384             | .430               | ( $\times 10^{-2}$ ) |  | .866             | .867               | ( $\times 10^{-2}$ ) |  | .432             | .430               | ( $\times 10^{-2}$ ) |  |  |
| 10        | .481             | .481               | ( $\times 10^{-2}$ ) | .142             | .178               | ( $\times 10^{-2}$ ) |  | .481             | .481               | ( $\times 10^{-2}$ ) |  | .179             | .178               | ( $\times 10^{-2}$ ) |  |  |
| 12        | .303             | .303               | ( $\times 10^{-2}$ ) | .579             | .864               | ( $\times 10^{-3}$ ) |  | .303             | .303               | ( $\times 10^{-2}$ ) |  | .880             | .864               | ( $\times 10^{-3}$ ) |  |  |
| 14        | .218             | .218               | ( $\times 10^{-2}$ ) | .285             | .521               | ( $\times 10^{-3}$ ) |  | .218             | .218               | ( $\times 10^{-2}$ ) |  | .533             | .521               | ( $\times 10^{-3}$ ) |  |  |
| 16        | .175             | .175               | ( $\times 10^{-2}$ ) | .170             | .375               | ( $\times 10^{-3}$ ) |  | .175             | .175               | ( $\times 10^{-2}$ ) |  | .386             | .375               | ( $\times 10^{-3}$ ) |  |  |

|           | Hybrid System 3  |                    |                      |                  |                    |                      |
|-----------|------------------|--------------------|----------------------|------------------|--------------------|----------------------|
|           | Sync             |                    |                      | Async            |                    |                      |
| $E_b/N_0$ | $\overline{P}_e$ | $\overline{P}_e^G$ |                      | $\overline{P}_e$ | $\overline{P}_e^G$ |                      |
| 8         | .938             | .927               | ( $\times 10^{-2}$ ) | .467             | .459               | ( $\times 10^{-2}$ ) |
| 10        | .539             | .526               | ( $\times 10^{-2}$ ) | .203             | .195               | ( $\times 10^{-2}$ ) |
| 12        | .351             | .337               | ( $\times 10^{-2}$ ) | 1.04             | .965               | ( $\times 10^{-3}$ ) |
| 14        | .258             | .244               | ( $\times 10^{-2}$ ) | .644             | .585               | ( $\times 10^{-3}$ ) |
| 16        | .211             | .197               | ( $\times 10^{-2}$ ) | .472             | .422               | ( $\times 10^{-3}$ ) |

**Table 3. Error Probability for Convolutional coded Systems (rate=0.5, constraint length=7)**

**(a) DS Systems ( $2K=24$ ,  $N=63$ )**

|           | System 1 |       |                       | System 2 |       |                       | System 3 |       |                       |
|-----------|----------|-------|-----------------------|----------|-------|-----------------------|----------|-------|-----------------------|
| $E_b/N_0$ | Sync     | Async |                       | Sync     | Async |                       | Sync     | Async |                       |
| 8         | 2.61     | .357  | ( $\times 10^{-6}$ )  | 2.61     | .357  | ( $\times 10^{-6}$ )  | 3.26     | .443  | ( $\times 10^{-6}$ )  |
| 10        | 3.51     | .188  | ( $\times 10^{-7}$ )  | 3.52     | .188  | ( $\times 10^{-7}$ )  | 4.89     | .268  | ( $\times 10^{-7}$ )  |
| 12        | 6.33     | .121  | ( $\times 10^{-8}$ )  | 6.34     | .121  | ( $\times 10^{-8}$ )  | 9.92     | .208  | ( $\times 10^{-8}$ )  |
| 14        | 16.3     | .117  | ( $\times 10^{-9}$ )  | 16.3     | .118  | ( $\times 10^{-9}$ )  | 28.7     | .248  | ( $\times 10^{-9}$ )  |
| 16        | 59.6     | .188  | ( $\times 10^{-10}$ ) | 59.7     | .188  | ( $\times 10^{-10}$ ) | 116.     | .485  | ( $\times 10^{-10}$ ) |

**(b) Hybrid Systems ( $K_h = 10$ ,  $2K_d = 24$ ,  $N = 63$ ,  $q = 100$ ,  $N_b = 100$ )**

|           | System 1 |       |                       | System 2 |       |                      | System 3 |       |                      |
|-----------|----------|-------|-----------------------|----------|-------|----------------------|----------|-------|----------------------|
| $E_b/N_0$ | Sync     | Async |                       | Sync     | Async |                      | Sync     | Async |                      |
| 8         | 4.09     | .452  | ( $\times 10^{-6}$ )  | 4.09     | .563  | ( $\times 10^{-6}$ ) | 5.01     | .687  | ( $\times 10^{-6}$ ) |
| 10        | 6.78     | .277  | ( $\times 10^{-7}$ )  | 6.78     | .401  | ( $\times 10^{-7}$ ) | 9.04     | .547  | ( $\times 10^{-7}$ ) |
| 12        | 15.5     | .217  | ( $\times 10^{-8}$ )  | 15.5     | .389  | ( $\times 10^{-8}$ ) | 22.5     | .604  | ( $\times 10^{-8}$ ) |
| 14        | 50.2     | .263  | ( $\times 10^{-9}$ )  | 50.2     | .607  | ( $\times 10^{-9}$ ) | 7.92     | .107  | ( $\times 10^{-8}$ ) |
| 16        | 224.     | .527  | ( $\times 10^{-10}$ ) | 22.4     | .156  | ( $\times 10^{-9}$ ) | 37.8     | .304  | ( $\times 10^{-9}$ ) |

**Table 4. Error Probability for Reed Solomon coded Systems (RS(32,16) over GF(2<sup>5</sup>))**

**(a) DS Systems (2K=24, N=63)**

|           | System 1 |       |                      | System 2 |       |                      | System 3 |       |                      |
|-----------|----------|-------|----------------------|----------|-------|----------------------|----------|-------|----------------------|
| $E_b/N_0$ | Sync     | Async |                      | Sync     | Async |                      | Sync     | Async |                      |
| 8         | 2.27     | .321  | ( $\times 10^{-2}$ ) | 2.27     | .321  | ( $\times 10^{-2}$ ) | 2.72     | .408  | ( $\times 10^{-2}$ ) |
| 10        | 31.6     | .749  | ( $\times 10^{-4}$ ) | 31.6     | .749  | ( $\times 10^{-4}$ ) | 4.53     | .123  | ( $\times 10^{-3}$ ) |
| 12        | 39.1     | .123  | ( $\times 10^{-5}$ ) | 39.1     | .124  | ( $\times 10^{-5}$ ) | 69.7     | .286  | ( $\times 10^{-5}$ ) |
| 14        | 613.     | .283  | ( $\times 10^{-7}$ ) | 614.     | .284  | ( $\times 10^{-7}$ ) | 1350     | .968  | ( $\times 10^{-7}$ ) |
| 16        | 1420     | .130  | ( $\times 10^{-8}$ ) | 1420     | .131  | ( $\times 10^{-8}$ ) | 3780     | .649  | ( $\times 10^{-8}$ ) |

**(b) Hybrid Systems ( $K_h = 10$ ,  $2K_d = 24$ ,  $N = 63$ ,  $q = 100$ ,  $N_b = 100$ )**

|           | System 1 |       |                      | System 2 |       |                      | System 3 |       |                      |
|-----------|----------|-------|----------------------|----------|-------|----------------------|----------|-------|----------------------|
| $E_b/N_0$ | Sync     | Async |                      | Sync     | Async |                      | Sync     | Async |                      |
| 8         | 3.25     | .417  | ( $\times 10^{-2}$ ) | 3.25     | .528  | ( $\times 10^{-2}$ ) | 3.78     | .650  | ( $\times 10^{-2}$ ) |
| 10        | 6.42     | .128  | ( $\times 10^{-3}$ ) | 6.42     | .213  | ( $\times 10^{-3}$ ) | 8.59     | .322  | ( $\times 10^{-3}$ ) |
| 12        | 121      | .306  | ( $\times 10^{-5}$ ) | 121      | .748  | ( $\times 10^{-5}$ ) | 18.9     | .145  | ( $\times 10^{-4}$ ) |
| 14        | 288      | .107  | ( $\times 10^{-6}$ ) | 288      | .412  | ( $\times 10^{-6}$ ) | 52.3     | .101  | ( $\times 10^{-5}$ ) |
| 16        | 9590     | .745  | ( $\times 10^{-8}$ ) | 961      | .452  | ( $\times 10^{-7}$ ) | 196      | .135  | ( $\times 10^{-6}$ ) |